

## IIB nine-branes

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**ABSTRACT:** We calculate the tensions of all half-supersymmetric nine-branes in IIB string theory. In particular, we point out the existence of a solitonic IIB nine-brane. We find that the D9-brane and its duality transformations parametrize a two-dimensional surface in a four-dimensional space.

**KEYWORDS:** D-branes, p-branes, Supergravity Models.

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## 1. Introduction

Using the supersymmetry algebra, there is a standard procedure to construct the supergravity multiplets of IIA and IIB supersymmetry. Naturally, the field content of these multiplets is such that there is an equal number of (on-shell) bosonic and fermionic degrees of freedom. However, it turns out that additional bosonic spacetime fields, which do not describe propagating degrees of freedom, can be added to these multiplets. They play an important role in describing the coupling of supergravity to branes.

An example of this phenomenon is the addition of a nine-form potential to the IIA supergravity multiplet [1] which couples to a D8-brane. Integrating out this potential leads to an integration constant that can be identified as the cosmological constant parameter of massive IIA supergravity [2]. Another example is the addition of a RR ten-form potential to the IIB supergravity multiplet that couples to a D9-brane [3]. From a string theory point of view, D9-branes take part in obtaining ten-dimensional type-I string theory from type-IIB. Indeed, in the closed sector the orientifold projection [4] generates a tadpole, and tadpole cancellation, i.e. cancellation of the overall RR charge and tension, requires the introduction of an open sector, corresponding to D9-branes. The ten-form potential corresponding to these branes does not have any field strength, and therefore there is no supergravity solution corresponding to the D9-brane. Nevertheless, the RR ten-form potential can consistently be included in the supersymmetry algebra, and its gauge and supersymmetry transformations formally determine the world volume action of the D9-brane, from which the open sector of the low-energy action of the type-I theory is obtained after truncation. In [5] it was shown that the IIB algebra can be extended in order to include the RR ten-form and a second ten-form potential. Somewhat surprisingly, it has been pointed out that these potentials cannot be related by an  $SL(2, \mathbb{R})$  transformation [6].

Recently, these issues were sorted out when it was shown that the ten-form that couples to the D9-brane transforms as a component of a quadruplet representation under the duality group  $SL(2, \mathbb{R})$  [7] whereas the second ten-form potential of [5] transforms as the component of a separate doublet representation.

Each of these ten-form potentials couples to a nine-brane. It is the purpose of this letter to calculate which of these nine-branes correspond to half-supersymmetric BPS objects and to calculate their tensions.

## 2. Brane tensions and BPS conditions

To calculate the tension of a single p-brane and to determine the BPS condition it is convenient to consider the leading terms of the full kappa-symmetric brane action, assuming that such an action exists. To be precise we consider (in static gauge) the Nambu-Goto term and the term that describes the coupling of a  $(p + 1)$ -form potential  $A_{(p+1)}$  to the p-brane:

$$\mathcal{L}_{\text{brane}} = \tau_{\text{brane}} \sqrt{-g} + \epsilon^{\mu_1 \dots \mu_{p+1}} A_{\mu_1 \dots \mu_{p+1}}. \quad (2.1)$$

Here  $\tau_{\text{brane}}$  is the brane tension which in general is a function of the scalars at hand (a dilaton for IIA and a dilaton plus axion for IIB).

We now consider the supersymmetry variation of the brane action (2.1), but keep only terms linear in the gravitino<sup>1</sup>. In this letter we restrict ourselves to the IIB theory. We will consider the IIA case in an upcoming work [10]. The relevant supersymmetry variations of the spacetime metric  $g_{\mu\nu}$  and the  $(p + 1)$ -form  $A_{(p+1)}$  are given by<sup>2</sup>

$$\delta g_{\mu\nu} = 2i\bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} + \text{h.c.}, \quad \delta A_{\mu_1 \dots \mu_{p+1}} \sim f \bar{\epsilon}\gamma_{[\mu_1 \dots \mu_p} \sigma \psi_{\mu_{p+1}]}, \quad (2.2)$$

where  $f$  is a function of the dilaton plus axion and  $\sigma$  is one of the three Pauli-matrices<sup>3</sup>. For a half-supersymmetric p-brane we require that the brane action is invariant under 16 of the 32 linear IIB supersymmetries (in addition there will be 16 nonlinear supersymmetries). For this to happen it is mandatory that the supersymmetry variation of the brane action is proportional to a projection operator that projects away half of the supersymmetry parameters. In general we find

$$\delta \mathcal{L}_{\text{brane}} \sim (\tau_{\text{brane}} \mathbb{1} + f \gamma_{01 \dots p} \sigma) \epsilon. \quad (2.3)$$

This variation is proportional to a projection operator provided

$$\tau_{\text{brane}} = f. \quad (2.4)$$

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<sup>1</sup>A similar variation was considered in [8] in the context of a supersymmetric Randall-Sundrum scenario, and was also discussed in [9].

<sup>2</sup>We work with real spinors in string frame, see section 6 of [7]. All spinors are two-component vectors, with each component being a 16-component Majorana-Weyl spinor.

<sup>3</sup>This is not the generic situation. Sometimes we are dealing, in the same supersymmetry variation, with two terms each containing a different Pauli matrix and a different dependence on the scalars. In that case the formulae below need a slight modification. As an example of such a situation, see the discussion of the D1-brane below.

We conclude that requiring supersymmetry in this sector not only determines the brane tension, via (2.4), but also fixes the BPS condition on the supersymmetry parameter, via (2.3) and (2.4). Note that, in order to have a projection operator in (2.3) we must have  $\sigma = \sigma_1$  or  $\sigma = \sigma_3$  for  $p = 1, 5, 9$  and  $\sigma = i\sigma_2$  for  $p = 3, 7$ .

Instead of terms linear in the gravitino, one can also consider terms linear in the dilatino. This requires varying the brane tension in front of the Nambu-Goto term. In all cases, except for nine-branes (see section 5) we find that all dilatino terms cancel provided that the same projection operator and the same brane tension is used that follows from requiring the cancellation of the gravitino terms. This provides a non-trivial check on our calculations.

### 3. Strings, three-branes and five-branes

As a key example we consider the fundamental string F1 and the Dirichlet brane D1. These branes form a doublet under  $SL(2, \mathbb{R})$  in which D1 is the S-dual of F1<sup>4</sup>.

In a formulation where the world volume vector field has been integrated out<sup>5</sup> for both the F1 and the D1 [11], the Nambu-Goto and Wess-Zumino terms are given by ( $\phi$  is the dilaton and  $\ell$  is the axion):<sup>6</sup>

$$\mathcal{L}_{D1} = \tau_{D1}\sqrt{-g} + \frac{1}{4}\epsilon^{\mu\nu}C_{\mu\nu}, \tag{3.1}$$

$$\mathcal{L}_{F1} = \tau_{F1}\sqrt{-g} + \frac{1}{4}\epsilon^{\mu\nu}B_{\mu\nu}. \tag{3.2}$$

Here  $C_{\mu\nu}$  and  $B_{\mu\nu}$  are two-form potentials that transform as a doublet under  $SL(2, \mathbb{R})$ . The relevant supersymmetry rules of these potentials are given by

$$\delta B_{\mu\nu} = 8i\bar{\epsilon}\sigma_3\gamma_{[\mu}\psi_{\nu]}, \quad \delta C_{\mu\nu} = -8ie^{-\phi}\bar{\epsilon}\sigma_1\gamma_{[\mu}\psi_{\nu]} + \ell\delta B_{\mu\nu}. \tag{3.3}$$

This can be used to determine the tensions and the BPS conditions of the F1- and D1-branes, as explained above. In this way we find for the fundamental string:

$$\tau_{F1} = 1, \quad \frac{1}{2}(\mathbb{1} + \sigma_3\gamma_{01})\epsilon = 0. \tag{3.4}$$

The analysis of the D1-brane is slightly more subtle because, due to the  $\ell\delta B_{\mu\nu}$  term in the variation of  $C_{\mu\nu}$ , the variation of the D1 Wess-Zumino term contains both terms with a  $\sigma_1$  and a  $\sigma_3$  matrix:

$$\delta\mathcal{L}_{D1} \sim (\tau_{D1}\mathbb{1} + (f\sigma_1 + g\sigma_3)\gamma_{01})\epsilon, \tag{3.5}$$

with  $f = -e^{-\phi}$  and  $g = \ell$ . Since  $\sigma_1$  and  $\sigma_3$  anticommute we find instead of (2.4):

$$\tau_{D1} = \sqrt{f^2 + g^2} = \sqrt{e^{-2\phi} + \ell^2}. \tag{3.6}$$

Our results so far are summarized in table 1. The table also contains the results for the 3- and 5-branes which can be derived similarly.

<sup>4</sup>In this paper we mean by S-duality the discrete  $\mathbb{Z}_2$  transformation.

<sup>5</sup>Integrating out the worldvolume vector field is optional. However, to preserve  $SL(2, \mathbb{R})$  symmetry one should do this for both or none of the two branes.

<sup>6</sup>In this letter all  $p$ -forms are real. With respect to [7] in some cases the  $p$ -form is multiplied by a factor of  $i$ . In the present case  $C = iC_{\text{old}}$ .

potential	brane	tension	projection operator
$C_{(2)}$	D1	$\sqrt{e^{-2\phi} + \ell^2}$	$\frac{1}{2}(\mathbb{1} + \frac{-e^{-\phi}\sigma_1 + \ell\sigma_3}{\sqrt{e^{-2\phi} + \ell^2}}\gamma_{01})$
$B_{(2)}$	F1	1	$\frac{1}{2}(\mathbb{1} + \sigma_3\gamma_{01})$
$C_{(4)}$	D3	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + i\sigma_2\gamma_{0123})$
$C_{(6)}$	D5	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + \sigma_1\gamma_{01\dots 5})$
$B_{(6)}$	NS5	$e^{-\phi}\sqrt{e^{-2\phi} + \ell^2}$	$\frac{1}{2}(\mathbb{1} + \frac{e^{-\phi}\sigma_3 + \ell\sigma_1}{\sqrt{e^{-2\phi} + \ell^2}}\gamma_{01\dots 5})$

**Table 1:** The two 2-, 4- and 6-form potentials of IIB supergravity and the corresponding branes in string frame.

We now consider a  $(p, q)$ -string (see [12]) where F1 denotes a  $(1, 0)$ -string and D1 a  $(0, 1)$ -string:

$$\mathcal{L}_{(p,q)} = \tau_{(p,q)} \sqrt{-g} + \frac{1}{4} \epsilon^{\mu\nu} (p B_{\mu\nu} + q C_{\mu\nu}). \tag{3.7}$$

Again we find in the variation of  $\mathcal{L}_{(p,q)}$  a combination of two Pauli matrices:

$$\delta\mathcal{L}_{(p,q)} \sim (\tau_{(p,q)} \mathbb{1} + ((p + \ell q)\sigma_3 - e^{-\phi} q \sigma_1)\gamma_{01}) \epsilon. \tag{3.8}$$

The tension now becomes

$$\tau_{p,q} = \sqrt{(p + \ell q)^2 + e^{-2\phi} q^2}, \tag{3.9}$$

which reproduces the tension formula of [12]. Using Einstein frame this tension formula can be rewritten in the manifest  $SL(2, \mathbb{R})$ -invariant form<sup>7</sup>

$$\tau_{(p,q)}^E = \sqrt{q^\alpha q^\beta \mathcal{M}_{\alpha\beta}}, \tag{3.10}$$

with

$$q^\alpha = \begin{pmatrix} q \\ p \end{pmatrix} \quad \text{and} \quad \mathcal{M} = e^\phi \begin{pmatrix} \ell^2 + e^{-2\phi} & \ell \\ \ell & 1 \end{pmatrix}. \tag{3.11}$$

#### 4. Seven-branes

Before considering nine-branes, it is instructive to first consider seven-branes. There are three eight-form potentials<sup>8</sup>  $C_{(8)}, D_{(8)}$  and  $B_{(8)}$  that transform as a triplet under  $SL(2, \mathbb{R})$ , see table 2. They correspond to the three seven-branes D7, I7 and  $\widetilde{D7}$ , where  $\widetilde{D7}$  is minus the S-dual of D7 and I7 changes sign under under S-duality. Note that all seven-branes have the same projection operator and hence correspond to the single 3-form (or, equivalently, 7-form) central charge in the IIB supersymmetry algebra. This shows that a single central charge that is invariant under S-duality may correspond to different branes that transform non-trivially under S-duality.

<sup>7</sup>Note that  $\tau_{\text{string}} = e^{-\frac{p+1}{4}\phi} \tau_{\text{Einstein}}$  for a  $p$ -brane.

<sup>8</sup>There is a constraint on the nine-form field strengths (see [13, 14, 7]), such that the three eight-forms describe the same two propagating degrees of freedom as the dilaton and axion.

potential	brane	tension	projection operator
$C_{(8)}$	D7	$e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + i\gamma_{01\dots 7}\sigma_2)$
$D_{(8)}$	I7	$\ell e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + i\gamma_{01\dots 7}\sigma_2)$
$B_{(8)}$	$\widetilde{D7}$	$e^{-3\phi} + \ell^2 e^{-\phi}$	$\frac{1}{2}(\mathbb{1} + i\gamma_{01\dots 7}\sigma_2)$

**Table 2:** The three eight-form potentials of IIB supergravity and the corresponding seven-branes.

Consider now a combination of seven-branes:

$$\mathcal{L}_{(p,r,q)} \sim \tau_{(p,r,q)} \sqrt{-g} + \epsilon^{\mu_1 \dots \mu_8} (p C_{\mu_1 \dots \mu_8} + r D_{\mu_1 \dots \mu_8} + q B_{\mu_1 \dots \mu_8}). \quad (4.1)$$

The D7-brane corresponds to  $(p, r, q) = (1, 0, 0)$ , the others accordingly. We find that in the supersymmetry variation the terms linear in the gravitino are proportional to

$$\delta \mathcal{L}_{(p,r,q)} \sim (\tau_{(p,r,q)} \mathbb{1} + i(p e^{-\phi} + r \ell e^{-\phi} + q(e^{-3\phi} + \ell^2 e^{-\phi})) \gamma_{01\dots 7} \sigma_2) \epsilon. \quad (4.2)$$

This is proportional to a projection operator provided that

$$\tau_{(p,r,q)} = |p e^{-\phi} + r \ell e^{-\phi} + q(e^{-3\phi} + \ell^2 e^{-\phi})|. \quad (4.3)$$

Using Einstein frame this formula can be written in manifest  $SL(2, \mathbb{R})$ -invariant form as follows:

$$\tau_{(p,r,q)}^E = |q^{\alpha\beta} \mathcal{M}_{\alpha\beta}|, \quad (4.4)$$

with

$$q^{\alpha\beta} = \begin{pmatrix} q & r/2 \\ r/2 & p \end{pmatrix}. \quad (4.5)$$

In contrast to strings we can impose an  $SL(2, \mathbb{R})$ -invariant constraint on the matrix  $q^{\alpha\beta}$ :

$$\det [q^{\alpha\beta}] = -\alpha^2 \quad \text{or} \quad pq - \frac{r^2}{4} = -\alpha^2, \quad (4.6)$$

for some  $\alpha$ . We see that the D7-brane and the  $\widetilde{D7}$ -brane belong to the  $\alpha = 0$  conjugacy class but that I7 belongs to the  $\alpha^2 > 0$  conjugacy classes. The constraint (4.6) defines co-dimension 1 surfaces in  $\mathbb{R}^{2,1}$ . For  $\alpha \neq 0$  they are hyperboloids and for  $\alpha = 0$  a cone. These hypersurfaces are homogeneous spaces<sup>9</sup>, and, therefore, all of them can be constructed as coset spaces  $SL(2, \mathbb{R})/H_\alpha$  where  $H_\alpha$  is the isotropy group for a given  $\alpha$ . For instance, for  $\alpha = 0$ , we find that  $H_0$  is the subgroup of matrices of the form

$$\Lambda = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \quad (4.7)$$

that shift the value of  $\ell$  by a real constant, which is isomorphic to  $\mathbb{R}$ .

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<sup>9</sup>We thank Patrick Meessen for discussions of this point.

potential	brane	tension	projection operator
$\mathcal{D}_{(10)}$	S9	$e^{-2\phi}$	$\frac{1}{2}(\mathbb{1} + \sigma_3)$
$\mathcal{E}_{(10)}$	$\widetilde{\text{S9}}$	$e^{-2\phi}\sqrt{e^{-2\phi} + \ell^2}$	$\frac{1}{2}\left(\mathbb{1} + \frac{-e^{-\phi}\sigma_1 + \ell\sigma_3}{\sqrt{e^{-2\phi} + \ell^2}}\right)$

**Table 3:** The doublet of 10-form potentials, their tensions and their projection operators.

We can use the constraint (4.6) to solve for  $r$  in terms of  $p$  and  $q$ :

$$r(p, q) = \pm 2\sqrt{pq + \alpha^2}. \tag{4.8}$$

This provides us with a set of  $(p, q)$  seven-branes that define a two-dimensional manifold. For  $\alpha^2 = 0$  this manifold is parametrized by

$$(p, r, q) = (b^2, 2bd, d^2), \quad b, d \in \mathbb{R}. \tag{4.9}$$

The  $\alpha^2 > 0$  and  $\alpha^2 < 0$   $(p, q)$  seven-branes form distinct conjugacy classes whose elements cannot be obtained by any  $\text{SL}(2, \mathbb{R})$  transformation of the D7-brane which belongs to the  $\alpha^2 = 0$  conjugacy classes. Nevertheless, for each  $\alpha$  they represent supersymmetric seven-branes whose solutions have been constructed [15]. Using a basis with  $r = 0$  and restricting ourselves to  $\alpha^2 = 0, \pm 1$  we can write a representative for each conjugacy class as follows:

$$\alpha^2 = 0 : \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \alpha^2 = -1 : \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \alpha^2 = 1 : \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \tag{4.10}$$

where the 3-vector indicates a vector with components  $(p, r, q)$ . This shows that elements of the  $\alpha^2 = 1$  and  $\alpha^2 = -1$  conjugacy class correspond to duality transformations of bound states of D7 branes with  $\widetilde{\text{D7}}$  branes and  $\widetilde{\widetilde{\text{D7}}}$  branes (anti S-dual D7-branes), respectively.

### 5. Nine-branes

We now consider the main topic of this letter: nine-branes. As explained in [7] the ten-form potentials of IIB supergravity organize themselves in a doublet and quadruplet representation of  $\text{SL}(2, \mathbb{R})$ . Note that the 10-form potential  $C_{(10)}$  that couples to the D9-brane is in the quadruplet representation. Using the supersymmetry rules given in [7] it is straightforward to determine the tensions and BPS conditions of the different nine-branes by requiring the cancellation of all terms linear in the gravitino in the supersymmetry variation of the different 9-brane actions. The results for the doublet and quadruplet are summarized in tables 3 and 4, respectively.

The discussion of the doublet of nine-branes is rather similar to that of the doublet of strings. We find that the tension of a  $(p, q)$  nine-brane is given by

$$\tau_{(p,q)} = e^{-2\phi}\sqrt{(p + \ell q)^2 + e^{-2\phi}q^2}. \tag{5.1}$$

potential	brane	tension $\tau$ and projection operator $P$
$C_{(10)}$	D9	$\tau = e^{-\phi}$ $P = \frac{1}{2}(\mathbb{1} + \sigma_1)$
$D_{(10)}$	-	$\tau = e^{-\phi} \sqrt{\frac{1}{9} e^{-2\phi} + \ell^2}$ $P = \frac{1}{2} \left( \mathbb{1} + \frac{\ell \sigma_1 + \frac{1}{3} e^{-\phi} \sigma_3}{\sqrt{\frac{1}{9} e^{-2\phi} + \ell^2}} \right)$
$E_{(10)}$	-	$\tau = e^{-\phi} \sqrt{\left(\frac{1}{3} e^{-2\phi} + \ell^2\right)^2 + \frac{4}{9} \ell^2 e^{-2\phi}}$ $P = \frac{1}{2} \left( \mathbb{1} - \frac{\left(\frac{1}{3} e^{-2\phi} + \ell^2\right) \sigma_1 + \frac{2}{3} \ell e^{-\phi} \sigma_3}{\sqrt{\left(\frac{1}{3} e^{-2\phi} + \ell^2\right)^2 + \frac{4}{9} \ell^2 e^{-2\phi}}} \right)$
$B_{(10)}$	$\widetilde{\text{D9}}$	$\tau = e^{-\phi} (e^{-2\phi} + \ell^2)^{3/2}$ $P = \frac{1}{2} \left( \mathbb{1} - \frac{\ell \sigma_1 + e^{-\phi} \sigma_3}{\sqrt{e^{-2\phi} + \ell^2}} \right)$

**Table 4:** The quadruplet of 10-form potentials, their tensions and their projection operators. Note that there is no half-supersymmetric nine-brane that couples to  $D_{(10)}$  or  $E_{(10)}$ .

In Einstein frame the tension is given by the manifest  $\text{SL}(2, \mathbb{R})$ -invariant tension-formula (3.10).

The discussion of the quadruplet of nine-branes is more subtle. We first consider  $(p, r, s, q)$ -branes

$$\mathcal{L}_{(p,r,s,q)} \sim \tau_{(p,r,s,q)} \sqrt{-g} + \epsilon^{\mu_1 \dots \mu_{10}} (p C_{\mu_1 \dots \mu_{10}} + r D_{\mu_1 \dots \mu_{10}} + s E_{\mu_1 \dots \mu_{10}} + q B_{\mu_1 \dots \mu_{10}}), \quad (5.2)$$

in which the  $(1, 0, 0, 0)$ -brane represents the D9-brane etc. We find that the tension of a  $(p, r, s, q)$ -brane is given by

$$\begin{aligned} \tau_{(p,r,s,q)} = & \left[ \left\{ e^{-\phi} p + \ell e^{-\phi} r - \left( \frac{1}{3} e^{-3\phi} + \ell^2 e^{-\phi} \right) s - \left( \ell^3 e^{-\phi} + \ell e^{-3\phi} \right) q \right\}^2 \right. \\ & \left. + \left\{ \frac{1}{3} e^{-2\phi} r - \frac{2}{3} \ell e^{-2\phi} s - \left( e^{-4\phi} + \ell^2 e^{-2\phi} \right) q \right\}^2 \right]^{1/2}. \end{aligned} \quad (5.3)$$

In Einstein frame the manifest  $\text{SL}(2, \mathbb{R})$ -invariant tension is given by

$$\tau_{(p,r,s,q)}^{\text{E}} = \sqrt{q^{\alpha\beta\gamma} q^{\delta\epsilon\zeta} \mathcal{M}_{\alpha\beta} \mathcal{M}_{\delta\epsilon} \mathcal{M}_{\gamma\zeta}}, \quad (5.4)$$

where

$$q^{222} = p, \quad q^{122} = -\frac{1}{3} r, \quad q^{112} = -\frac{1}{3} s, \quad q^{111} = q. \quad (5.5)$$

So far we have only achieved the cancellation of the gravitino terms. These cancellations merely serve to derive the tension formulae and the BPS conditions. A first nontrivial check on supersymmetry is the cancellation of the dilatino terms. Unlike the previous cases we find that these dilatino terms only cancel provided that

$$3qr + s^2 = 0, \quad 3ps + r^2 = 0, \quad 9pq - rs = 0. \quad (5.6)$$



Note that these constraints are satisfied by the D9-brane and its S-dual but *not* by the other two 9-branes. For generic points  $(p, r, s, q)$  the last constraint follows from the first two but not for special cases. Therefore, it cannot be omitted. For instance, the points  $(p, 0, 0, q)$  solve the first two constraints for general  $p, q$  but solving the third constraint requires  $p = 0$  or  $q = 0$ .

To understand the  $SL(2, \mathbb{R})$  properties of the constraints (5.6) it is convenient to introduce the matrix  $Q^{\alpha\beta} \equiv q^{\alpha\gamma\delta} q^{\beta\epsilon\zeta} \epsilon_{\gamma\epsilon} \epsilon_{\delta\zeta}$  or

$$Q^{\alpha\beta} = \frac{1}{9} \begin{pmatrix} 2(3qr + s^2) & 9pq - rs \\ 9pq - rs & 2(3ps + r^2) \end{pmatrix}. \quad (5.7)$$

The constraints (5.6) can then be written as  $Q^{\alpha\beta} = 0$  and transform manifestly as a triplet of  $SL(2, \mathbb{R})$ . Using the constraints (5.6) to solve for  $r, s$  in terms of  $p, q$  we end up with a set of  $(p, q)$  9-branes that define a two-dimensional manifold in a four-dimensional space parametrized by

$$(p, r, s, q) = (d^3, -3bd^2, -3b^2d, b^3), \quad b, d \in \mathbb{R}. \quad (5.8)$$

This manifold can be identified as the  $SL(2, \mathbb{R})$  orbit of the D9-brane. It has the isotropy group (4.7). We thus end up with the same homogeneous space that we encountered for the orbit of the D7-brane. Unlike the case of seven-branes there are no other conjugacy classes of half-supersymmetric nine-branes.

Finally, we consider the "quantized" duality group  $SL(2, \mathbb{Z})$ . We assume that the brane charges are quantized. We first consider the linear doublet. The orbit of the S9-brane is given by:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}, \quad ad - bc = 1. \quad (5.9)$$

For any pair  $a, c$  of co-prime integers<sup>10</sup> we can use the extended Euclidean algorithm to solve for integers  $b$  and  $d$ . This shows that if we restrict the duality group to  $SL(2, \mathbb{Z})$ , all branes are in the orbit of the S9-brane. Note that the same argument applies to  $(p, q)$ -strings and  $(p, q)$  5-branes. The case of  $(p, q)$  7-branes, where different classes of constraints appear, is more complex and was treated in [16].

We next consider the non-linear doublet, where we find a similar result: all 9-branes lie in the  $SL(2, \mathbb{Z})$  orbit of a single D9-brane, if we do not count multiple branes of the same type as independent. This can be seen as follows<sup>11</sup>. All 9-branes lie in the  $SL(2, \mathbb{R})$  orbit of the D9-brane, given by the charges  $(d^3, -3bd^2, -3b^2d, b^3)$ . To verify our claim we have to check that all  $SL(2, \mathbb{R})$  transformations of the D9-brane which lead to integer charges are also elements of  $SL(2, \mathbb{Z})$ . To obtain integer charges, we must have  $d^3 = u$ ,  $d^2b = v$  for some integers  $u, v$ . If  $d$  and  $b$  are not integers themselves, then without loss of generality this implies  $b = nd$  for some integer  $n$ <sup>12</sup>. Any transformation of that kind, however, leads to a

<sup>10</sup>If  $a, c$  are not co-prime we consider them as multiple copies of a fundamental brane, see [12].

<sup>11</sup>A similar analysis, with a similar result, can be made of the  $\alpha^2 = 0$  conjugacy class of seven-branes, which contains the D7-brane.

<sup>12</sup>If only one of them is not an integer, then we do not get integer charges and if both are integers, then there is either an  $SL(2, \mathbb{Z})$  transformation which generates this brane from the D9-brane, or we are dealing with a multiple brane.

brane	D1	F1	D3	D5	NS5	D7	$\widetilde{D7}$	D9	$\widetilde{D9}$	S9	$\widetilde{S9}$
tension	$g_s^{-1}$	1	$g_s^{-1}$	$g_s^{-1}$	$g_s^{-2}$	$g_s^{-1}$	$g_s^{-3}$	$g_s^{-1}$	$g_s^{-4}$	$g_s^{-2}$	$g_s^{-3}$

**Table 5:** The tensions of the basic half-supersymmetric IIB branes, with vanishing axion. In the case of seven-branes we have not indicated the half-supersymmetric branes corresponding to the  $\alpha^2 > 0$  and  $\alpha^2 < 0$  conjugacy classes of  $SL(2, \mathbb{R})$ .

”multiple” brane with charges  $u(1, -3n, -3n^2, n^3)$ , which we do not consider independent, unless  $u = d = 1, b = n$ . In the latter case the brane is connected to the D9-brane by a  $SL(2, \mathbb{Z})$  transformation<sup>13</sup>. This verifies our claim.

## 6. Discussion

In this work we have determined the tensions of all half-supersymmetric branes of IIB string theory, including a linear and nonlinear doublet of  $(p, q)$  nine-branes. A brief summary of the results is given in table 5.

There are some surprises for the nine-branes. The standard D9-brane is part of a nonlinear doublet. This nonlinear doublet is expected to play a role in the construction of strings with sixteen supercharges, along the lines of refs. [17–19].<sup>14</sup> We find an additional linear doublet of nine-branes that contains a S9-brane whose tension scales as  $g_s^{-2}$ , i.e. a soliton. An obvious question to ask is: what is the world-volume dynamics of the S9-brane? We hope to report in this direction in a forthcoming paper [10]. The precise role of the S9-brane in IIB string theory is still unclear. Perhaps it has a role to play in the recently proposed non-perturbative open  $SO(32)$  heterotic string construction of [20].<sup>15</sup>

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<sup>13</sup>To construct this transformation, one can determine the values of  $a, c$  with the extended Euclidean algorithm, similarly to the S9-brane case.

<sup>14</sup>Observe, however, that in these papers the D9-brane was assumed to belong to a linear doublet together with the  $\widetilde{D9}$  (there named NS9), which was shown to have a tension proportional to  $g_s^{-4}$  in the string frame. We have shown here that this doublet is non-linear, with the corresponding potentials belonging to a quadruplet.

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